

OPTIMIZATION OF BUILDING WALL TO CONTROL INDOOR TEMPERATURE USING MATLAB SOFTWARE AND FINITE DIFFERENCE METHOD

Authors and Affiliation

Dinku Seyoum Zeleke (M.Sc. in thermal engineering), (**principal**), Addis Ababa Science and Technology University (P.Box 16417), University president, Ethiopia.

Muluwork Kasay (PhD in thermal engineering), (**Co-worker**), Bahir Dar University institute of technology, Ethiopia.

Abstract:

The complex nature of dynamic heat transfer through building parts is in need of computational analysis instead of applying simple mathematical techniques. An explicit finite difference method is applied to analyze a model wall of a building. And further computer simulation for results is done using MATLAB software by giving emphasis for parameters that can affect indoor atmospheric conditions. The result of the analysis has showed that, to obtain a favorable indoor temperature of 298k an optimum wall thickness of 25cm is required. The results from the numerical analysis have showed insignificant deviation of value around 5% from the actual analytical methods of analysis. Finally the research has indicated that in using PCM, the heat storage capacity of the wall will increase by 70% with a corresponding increase in 50% of the cost effectiveness in building construction.

Key words: Numerical method, Thickness of wall, efficiency and Temperature

Introduction:

The importance of analyzing thermal (energy and comfort) performance in building design is growing, because of the increasing awareness of the role of energy usage in building life cycle costs and environmental impacts and the role of indoor conditions in the personnel's productivity. However, thermal analysis is still often done using simple static calculations or even by statistical estimates. Accurate enough dynamic thermal simulation software's have been available already for decades [4]. The main barrier of wider usage of dynamic thermal analysis methods has been the required big manual input work [7]. Most of the building element specific information needed in thermal simulation is described in the building information model. A researcher is planning in obtaining safe indoor condition of building. The simulation software is a freely available, user-friendly two-dimensional heat transfer model for analyzing the impacts of thermal energy in building components [1]. The researcher begins by describing simulation software as a tool for analyzing individual building components as well as envelope assemblies. The goal of this study to explore different options in modeling site built products and their effects on energy performance of the whole building. The building simulation takes into account the interaction of different building components, occupancy, schedules and lighting systems [8].

Many simulation programs have been developed to estimate energy performance by different scholars. To mention few of them that contributes for the integration of phase change material to building to create naturally air-conditioned room, such as Samira Haghshenaskashani and Hadi Pashdarshahri in their title "Simulation of Thermal Storage Phase Change Material in Buildings", they investigated the application of commercial phase change material to building to control indoor temperature to minimize heating and cooling cost.

Mathematical Modeling and Programming:

The model is based on the finite difference approach; each wall is divided in several layers, which allows the modeling of multi-layer walls. The zone being modeled is a living room with 100m² floor area and four exterior walls. All exterior walls are (4 m) are facing west, east, south and north. The window area (shading factor 55%) is 20% of the exterior wall area. The opaque part of the walls has a U-value of 0.266 W/m².K (R-21), while the double pane windows have a U-value of 0.95W/m².K (R-6). The ceiling insulation level amounts to a U-value of 0.17 W/m².K(R-32).The wall structure contains four layers; such as exterior finish, blocket, phase change material and interior finish.

Assumptions

To study the system the following assumptions are made:

- ✓ The temperature variation is two dimensional (across width and depth directions only)
- ✓ The ambient temperature (T_{amb}) and solar heat flux (q_s) are the functions of time over the day;
- ✓ The material properties are constant within limited distance in x axis direction but full constant in y direction ;
- ✓ Inside and outside heat transfer coefficients are constant;
- ✓ Radiation heat exchange within the room is neglected

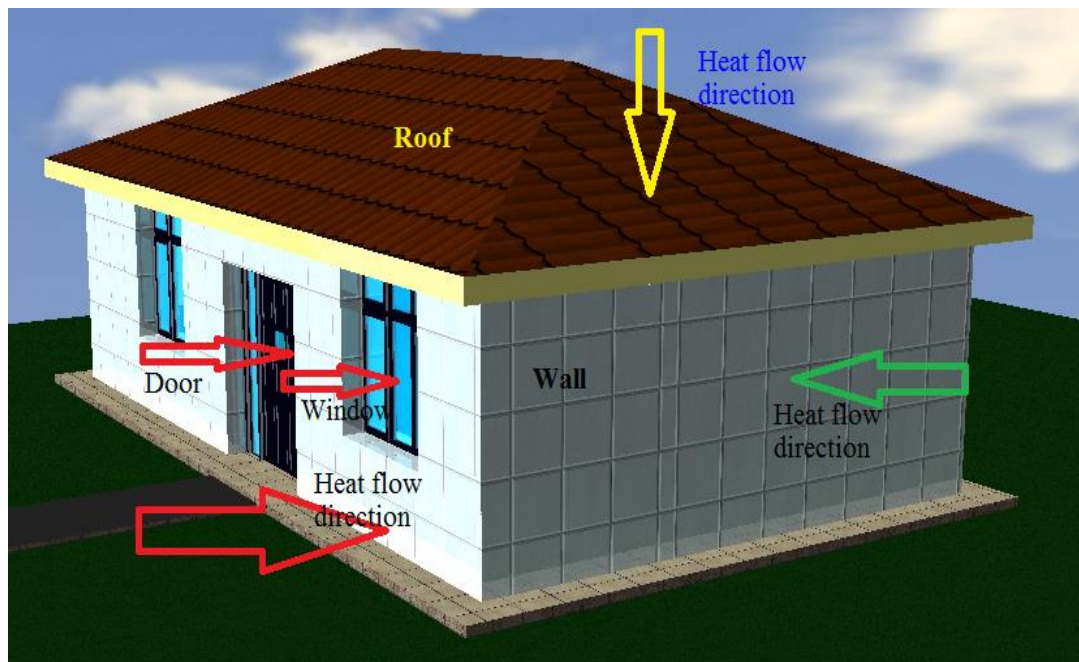


Figure 1: Model house for design

The house has basically three components such as roof, wall and floor . And heat enter and leave into and out of the room through this componets by means of radiation, conduction and convection . But most of the time one mode of heat transfer is dominant.In our case the dominant is conduction . Therefore it has

been limited in its scope only to simulation of wall heat distribution by conduction. And the analysis is done using finite difference method in the preceding sections.

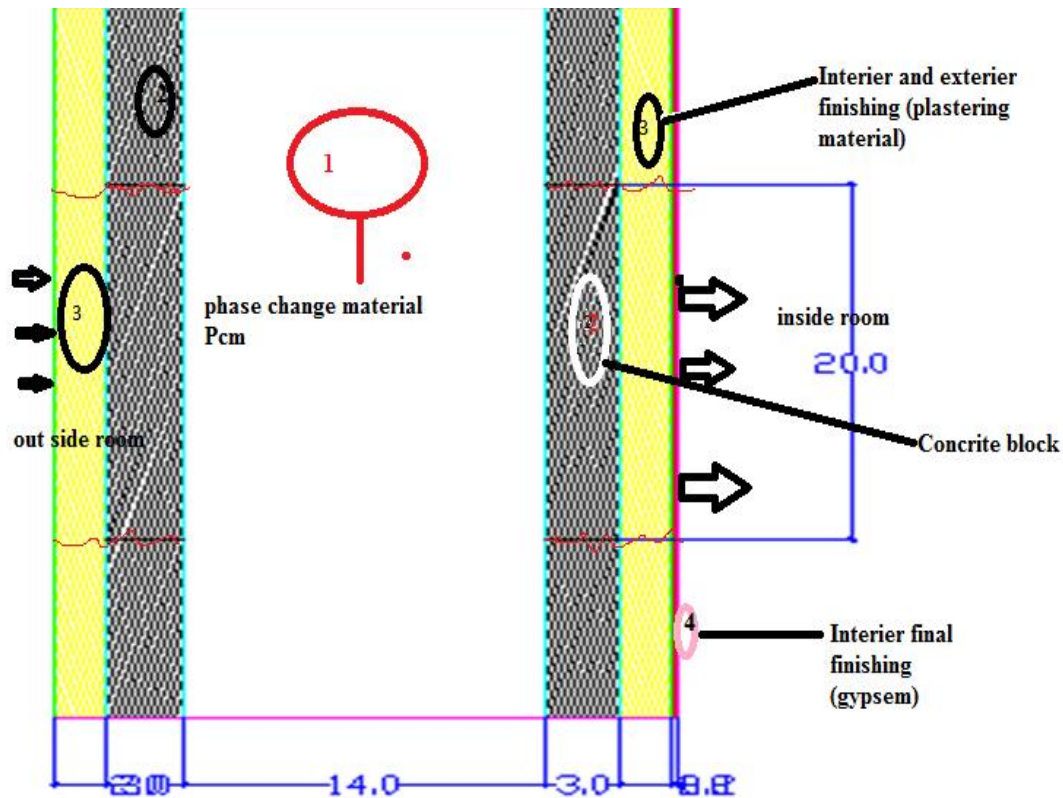


Figure 2: Side Section View of the Wall

Figure 2 shows the different layers of wall for thermal analysis and modeling of the system. The layers are differentiated with different color and hatch. The central layer is PCM as shown above at the extreme ends interior and exterior finishings are there.

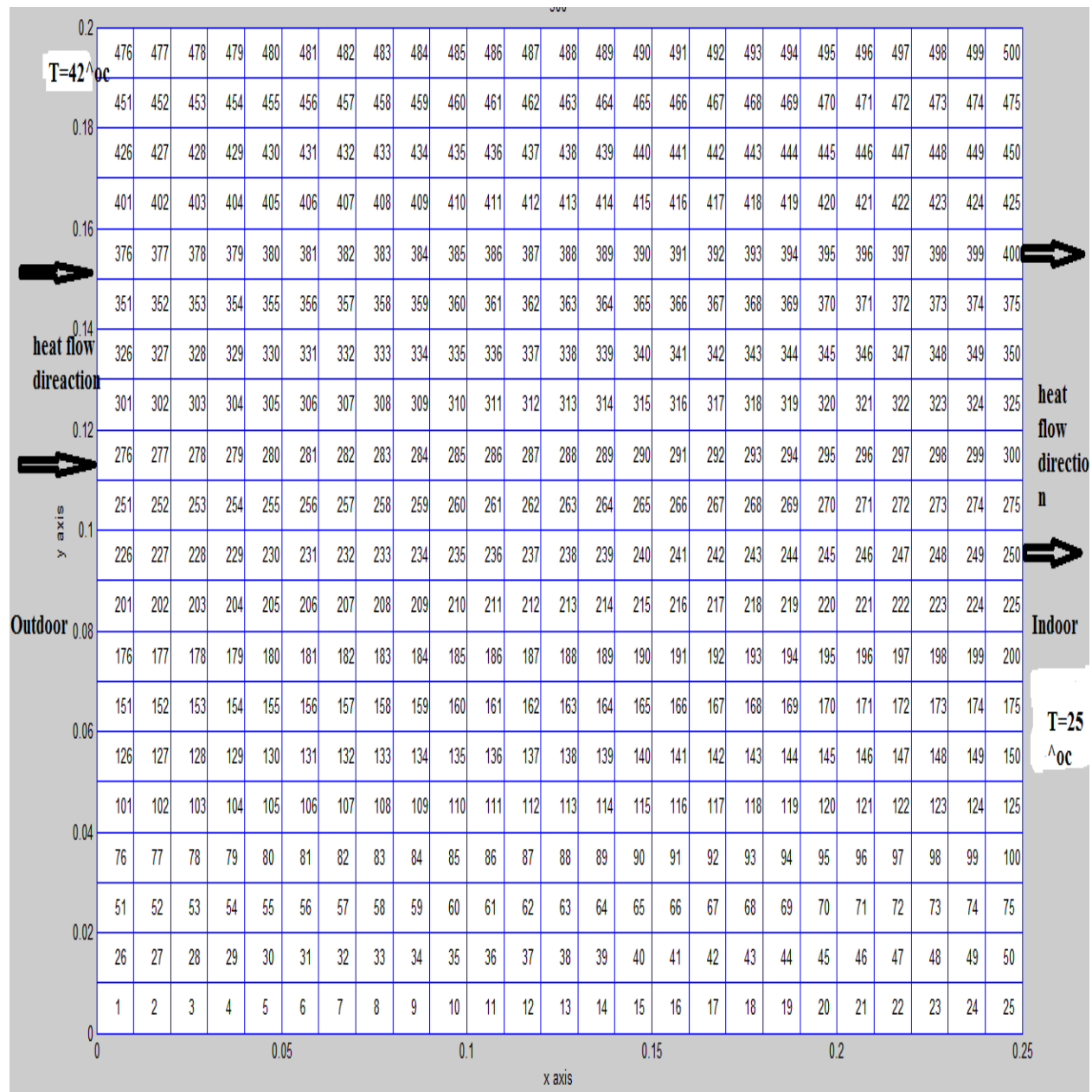


Figure 3: For finite difference analysis meshes from MATLAB

Figure 3 is element representation using number star by MATLAB and putting boundary condition and heat flow direction. There are 500 elements and 546 nodes. Each node is at different temperatures.

General heat conduction equation

$$\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + Q_{gene} \quad (1)$$

By neglecting heat generation and considering two dimensional

$$\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \quad (2)$$

Initial condition

$$\text{At, time} = 0, T = T_i, T(x,0) = T_i \text{ and } T(y,0) = T_i$$

Boundary condition

At $x=0$ and $y=0$ to $y-1$

$$\left. \frac{\partial T(0,t)}{\partial x} \right|_{x=0} + \left. \frac{\partial T(y,t)}{\partial y} \right|_y = 0 \quad (3)$$

At $x=x-1$ and $y=0$ to $y-1$

$$\left. \frac{\partial T(x,t)}{\partial x} \right|_{x=x-1} + \left. \frac{\partial T(y,t)}{\partial y} \right|_y = h[T(x,y,t) - T_\infty] \quad (4)$$

Two dimensional finite difference formulation of boundary condition (Explicit method)

✓ **Interior node**

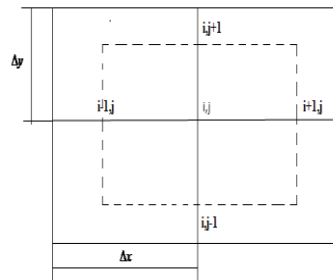


Figure 4: Center Node Representations

Now invoking energy balance equation

$$E_{in} + E_g - E_{out} = \Delta E_{st} \quad (5)$$

There is no heat generation, therefore E_g is equal to zero and stored energy is equal to the time gradient of temperature. In and out flow energies are the rate of heat energy as shown below.

$$q_{(m-1) \rightarrow (m,n)} = (K(\Delta y.1)) \frac{T_{m-1,n}^p - T_{m,n}^p}{\Delta x} \quad (6)$$

$$q_{(m+1) \rightarrow (m,n)} = (K(\Delta y.1)) \frac{T_{m+1,n}^p - T_{m,n}^p}{\Delta x} \quad (7)$$

$$q_{(m,n+1) \rightarrow (m,n)} = (K(\Delta x.1)) \frac{T_{m,n+1}^p - T_{m,n}^p}{\Delta x} \quad (8)$$

$$q_{(m,n-1) \rightarrow (m,n)} = (K(\Delta x.1)) \frac{T_{m,n-1}^p - T_{m,n}^p}{\Delta x} \quad (9)$$

$$\Delta E_{st} = \rho * (\Delta x * \Delta y * 1) C \left(\frac{T_{m,n}^{p+1} - T_{m,n}^p}{\Delta t} \right) \quad (10)$$

$$T_{m-1,n}^p + T_{m+1,n}^p + T_{m,n+1}^p + T_{m,n-1}^p - 4T_{m,n}^p = \frac{\rho C}{K} \frac{\Delta x^2}{\Delta t} (T_{m,n}^{p+1} - T_{m,n}^p) \quad (11)$$

$$\text{Let } F_o = \frac{\rho C}{K} \frac{\Delta x^2}{\Delta t} \text{ then} \quad (12)$$

$$T_{m,n}^{p+1} = F_o (T_{m-1,n}^p + T_{m+1,n}^p + T_{m,n+1}^p + T_{m,n-1}^p) + (1 - 4F_o) T_{m,n}^p \quad (13)$$

The above e Equation13 is used for all interior nodes of the wall nodal temperature calculation.

Stability criteria

$$\left. \begin{aligned} -4F_o &\geq 0 \\ -4F_o &\geq -1 \\ 4F_o &\leq 1 \\ F_o &\leq \frac{1}{4} \end{aligned} \right\} \quad (14)$$

✓ Node at plane surface with convection

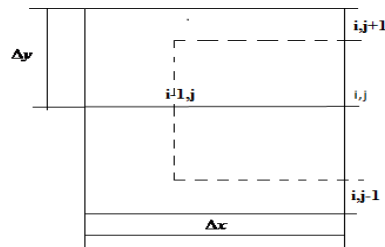


Figure 5: Side Node Representations

In the case of side nodes only inflow energy is there, therefore we equate inflow energy to stored energy.

$$\dot{E}_{in} = \Delta \dot{E}_{st} \quad (15)$$

$$q_{\infty,(m,n)} = K(\Delta y.1)(T_{\infty} - T_{m,n}^p) \quad (16)$$

$$q_{m-1,n \rightarrow (m,n)} = K(\Delta y.1) \frac{(T_{m-1,n}^p - T_{m,n}^p)}{\Delta x} \quad (17)$$

$$q_{m,n-1 \rightarrow (m,n)} = K \left(\frac{\Delta x}{2} \cdot 1 \right) \frac{(T_{m,n-1}^p - T_{m,n}^p)}{\Delta x} \quad (18)$$

$$q_{m,n+1 \rightarrow (m,n)} = K \left(\frac{\Delta x}{2} \cdot 1 \right) \frac{(T_{m,n+1}^p - T_{m,n}^p)}{\Delta x} \quad (19)$$

$$\Delta E_{st} = \rho C \cdot \frac{1}{2} \Delta x^2 \cdot 1 \frac{T_{m,n}^{p+1} - T_{m,n}^p}{\Delta t} \quad (20)$$

$$2T_{m-1,n}^p + T_{m,n+1}^p + T_{m,n-1}^p - 4T_{m,n}^p + \frac{2h\Delta x}{K}(T_\infty - T_{m,n}^p) = 2\rho C \frac{1}{2} \frac{\Delta x^2}{K\Delta t} \left(\frac{T_{m,n}^{p+1} - T_{m,n}^p}{1} \right) \quad (21)$$

$$F_o(2T_{m-1,n}^p + T_{m,n+1}^p + T_{m,n-1}^p) - 4F_o T_{m,n}^p + 2Bi \cdot F_o(T_\infty - T_{m,n}^p) = T_{m,n}^{p+1} \quad (22)$$

The above equation 22 is used for all side nodes of the wall to know nodal temperature.

Stability criteria

$$\left. \begin{aligned} 1 - 4F_o - 2BiF_o &\geq 0 \\ -4F_o - 2BiF_o &\geq -1 \\ 4F_o + 2BiF_o &\leq 1 \\ 2F_o(2 + Bi) &\leq 1 \\ F_o(2 + Bi) &\leq \frac{1}{2} \end{aligned} \right\} \quad (23)$$

✓ **Node at exterior corner with convection**

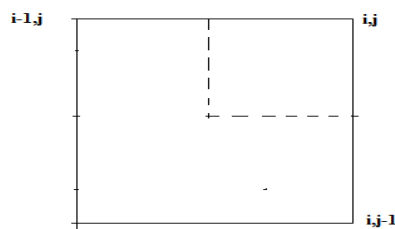


Figure 6: Corner Node Representation

In the case of corner nodes only inflow energy is there, therefore we equate inflow energy to stored energy.

$$\dot{E}_{in} = \Delta \dot{E}_{st} \quad (24)$$

$$\Delta E_{st} = \rho C \cdot \frac{1}{4} \Delta x^2 \cdot 1 \frac{T_{m,n}^{p+1} - T_{m,n}^p}{\Delta t} \quad (25)$$

$$q_{\infty,(m,n)} = K(\Delta x \cdot 1)(T_\infty - T_{m,n}^p) \quad 2.29$$

$$q_{m-1,n \rightarrow (m,n)} = K \left(\frac{\Delta y}{2} \cdot 1 \right) \frac{(T_{m-1,n}^p - T_{m,n}^p)}{\Delta x} \quad (26)$$

$$q_{m,n-1 \rightarrow (m,n)} = K \left(\frac{\Delta x}{2} \cdot 1 \right) \frac{(T_{m,n-1}^p - T_{m,n}^p)}{\Delta x} \quad (27)$$

$$\frac{K}{2} (T_{m-1,n}^p - T_{m,n}^p) + \frac{K}{2} (T_{m,n-1}^p - T_{m,n}^p) + h \Delta x (T_{\infty}^p - T_{m,n}^p) = \rho C \frac{1}{4} \frac{\Delta x^2}{\Delta t} \frac{T_{m,n}^{p+1} - T_{m,n}^p}{\Delta t} \quad (28)$$

$$T_{m-1,n}^p - 2T_{m,n}^p + T_{m,n-1}^p + \frac{2h\Delta x}{k} (T_{\infty}^p - T_{m,n}^p) = \rho C \frac{1}{2} \frac{\Delta x^2}{K \Delta t} \frac{T_{m,n}^{p+1} - T_{m,n}^p}{\Delta t} \quad (29)$$

$$T_{m,n}^{p+1} = T_{m,n}^p + (1 - 4F_o - 4BiF_o) * T_{m,n}^p + 2F_o (T_{m-1,n}^p + T_{m,n-1}^p + 2BiT_{\infty}^p) \quad (30)$$

The above equation is used for all 4 corner nodes, to obtain the temperature at those corners.

Stability criteria

$$\left. \begin{aligned} 1 - 4F_o - 4BiF_o &\geq 0 \\ -4F_o - 4BiF_o &\geq -1 \\ 4F_o + 4BiF_o &\leq 1 \\ 4F_o(1 + Bi) &\leq 1 \\ F_o(1 + Bi) &\leq \frac{1}{4} \end{aligned} \right\} \quad (31)$$

There are four interfaces and there thermal properties are the average of the two material properties. The analysis is done individually and assembled finally.

Developing simultaneous equation

For center nodes stability is a criterion for explicit method is $F_o = \frac{\rho C}{K} \frac{\Delta x^2}{\Delta t} \leq \frac{1}{4}$

$$\left. \begin{aligned} T_{28}^{p+1} &= (1 - 4F_o)T_{28}^p + F_o(T_{27}^p + T_{29}^p + T_{54}^p + T_2^p) \\ T_{29}^{p+1} &= (1 - 4F_o)T_{29}^p + F_o(T_{28}^p + T_{30}^p + T_{55}^p + T_3^p) \\ &\vdots \\ &\vdots \\ &\vdots \\ T_{519}^{p+1} &= (1 - 4F_o)T_{519}^p + F_o(T_{518}^p + T_{520}^p + T_{546}^p + T_{433}^p) \end{aligned} \right\} \quad (32)$$

The above Equations show the simultaneous solution for all center nodes. To build system matrix combine side, corner and center node equation together. Then the global matrix is developed in the following manner:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 2Fo & 1-4Fo & 2Fo & 0 & 0 & 0 \\ 0 & 2Fo & 1-4Fo & 2Fo & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 2Fo & 1-4Fo & 2Fo \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} T_{28}^{p+1} \\ T_{29}^{p+1} \\ \vdots \\ \vdots \\ T_{519}^{p+1} \end{Bmatrix} = \begin{Bmatrix} T_{28}^p \\ T_{29}^p \\ \vdots \\ \vdots \\ T_{519}^p \end{Bmatrix} \quad (33)$$

This global matrix has been developed by invoking tridiagonal matrix principle.

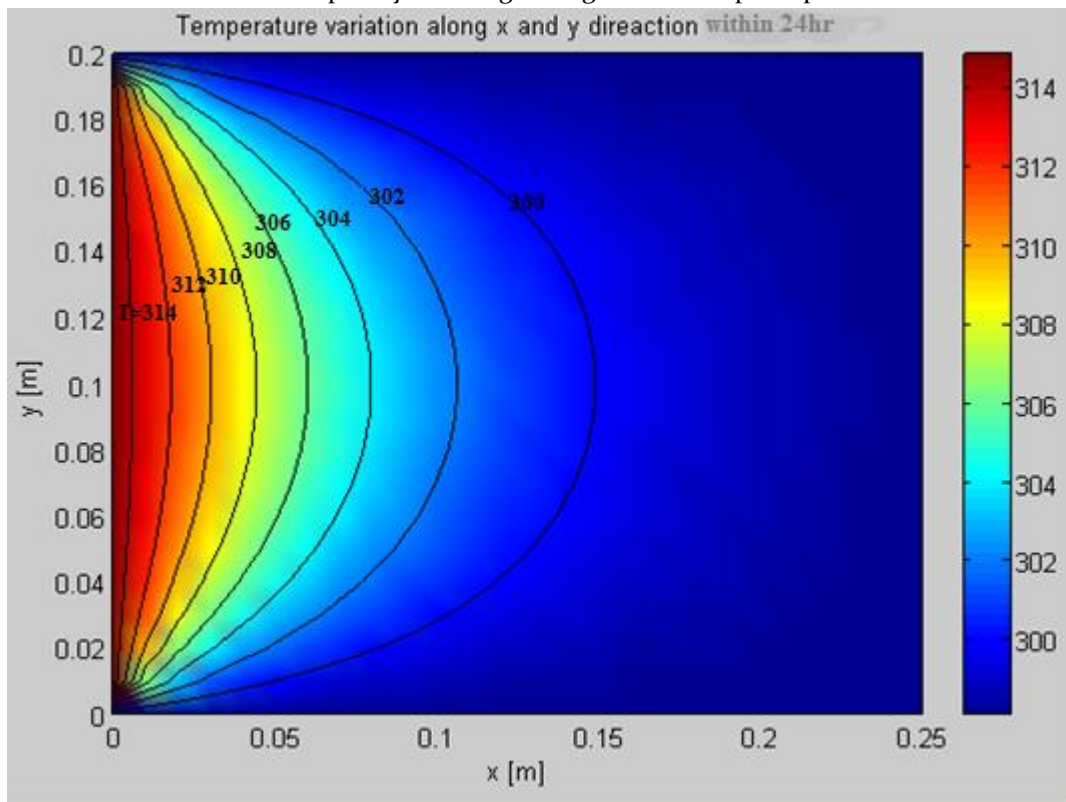


Figure 7: Temperature Variation along x and y Direction

Figure 7 shows the temperature variation along y and x direction and the graph was obtained using finite difference method and imposing boundary condition to the MATLAB7. The temperature profile is straight line at higher temperature. But when the temperature decreases this variation is changed to parabolic shape. This parabolic shape shows the temperature variation is slowly decreasing. Therefore the optimum thickness in which the comfortable temperature obtained is 25cm. The values of the temperature given in the graph are all in degree Kelvin (K).

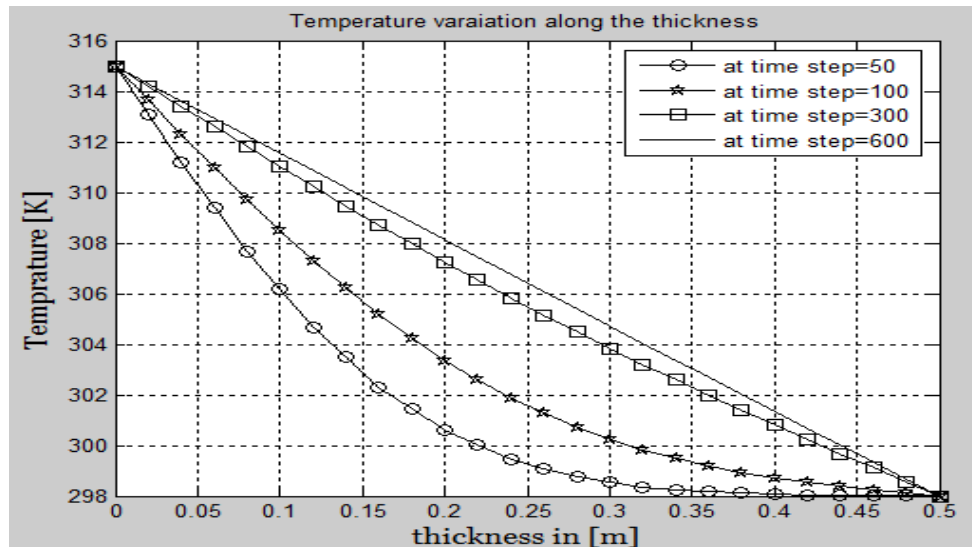


Figure 8: Temperature Variations through the Wall at Different Time Step

Figure 8 show temperature variations with thickness of the wall at different time step using finite difference method. From the graph above we can observe, the temperature variation is very high when the time increases and at time equal to zero or at initial time temperature is nearly constant. Therefore can be concluded that the temperature variation through the wall thickness is not linear instead it is half parabolic curve. And at small time step the solution is stable and fails under the optimum length.

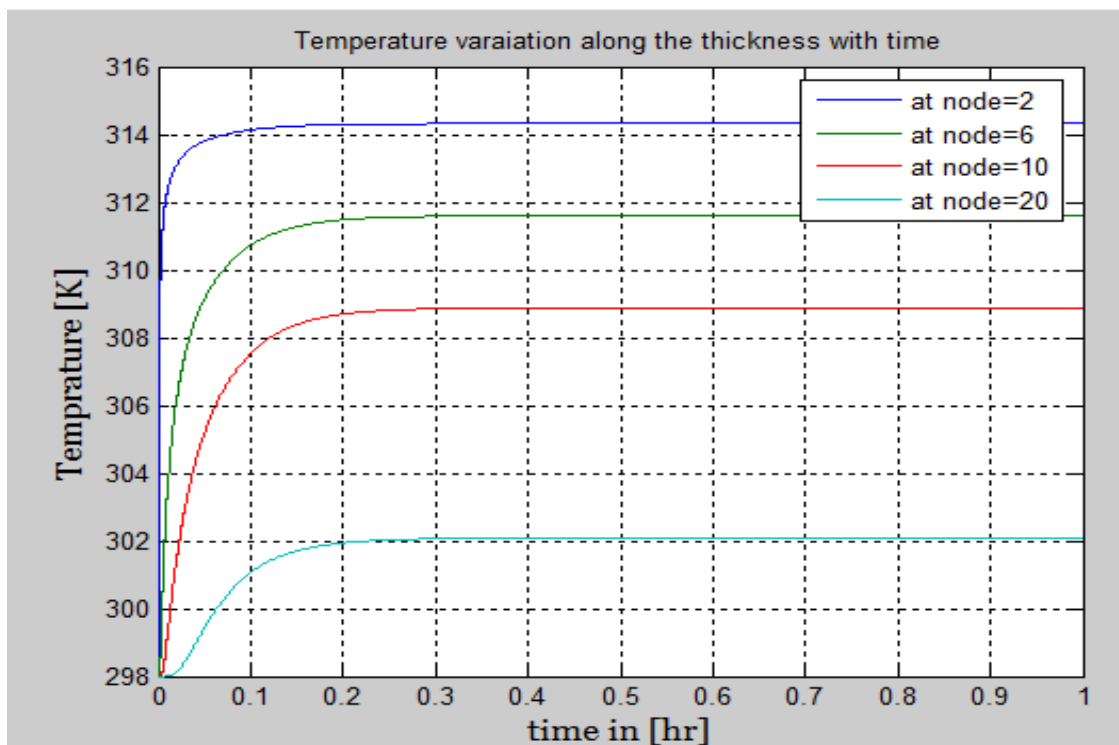


Figure 9: Temperature Variations with Time for Different Nodes

Figure 9 shows the variation of temperature at different node with time using finite difference method. And from figure above we observed the following things: for node 20 the temperature variation within 0.2hr time is 4°C, for node 10 the temperature variation within 0.2hrs time is 8°C, for node 6 the temperature variation within 0.2hr time is 11°C and for node 2 the temperature variation within 0.2hr time is 4°C. Therefore this shows that, the temperature change at inside surface of wall and outside surface of the wall is affected by different factors such as environmental condition, type of wall material and inside room appliances. Therefore the temperature variation is maximum at inside and outside surface nodes.

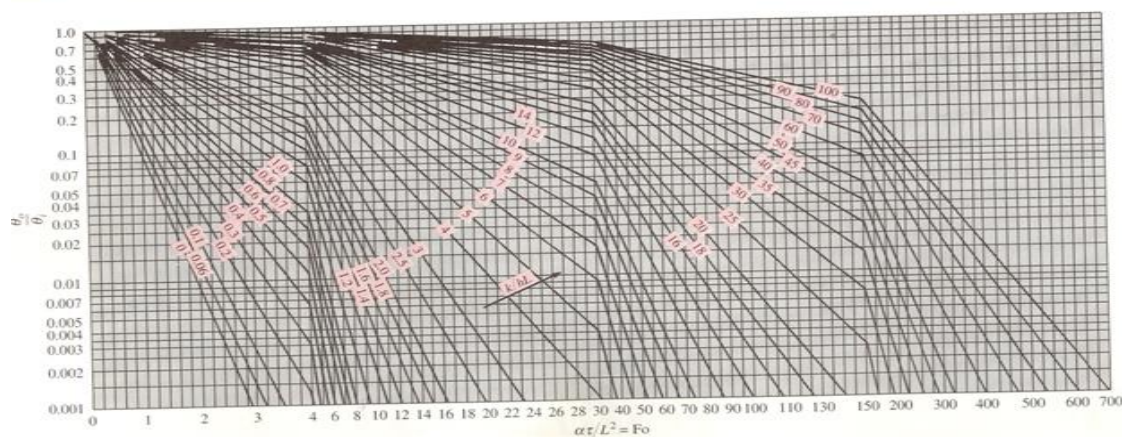
Conclusion:

The storage density of PCM-integrated wall is much higher, which allows providing the necessary storage in a relatively thin layer. Longer cycles between charge and discharge can be mastered with higher thermal storage capacity. The additional storage capacity can be provided by multiple layers or thicker layers of PCM-integrated wall. In conclusion, the use of PCM will result a 70% increase in heat storage capacity of the wall.. Therefore to control the cooling and heating of the wall instead of using air-conditioning system it is so better to use phase change materials to eliminate the operation cost of air-conditioning system that are used to cool or heat a building. The basic numerical analysis is done using an explicit finite difference method the results are obtained from MATLAB program.

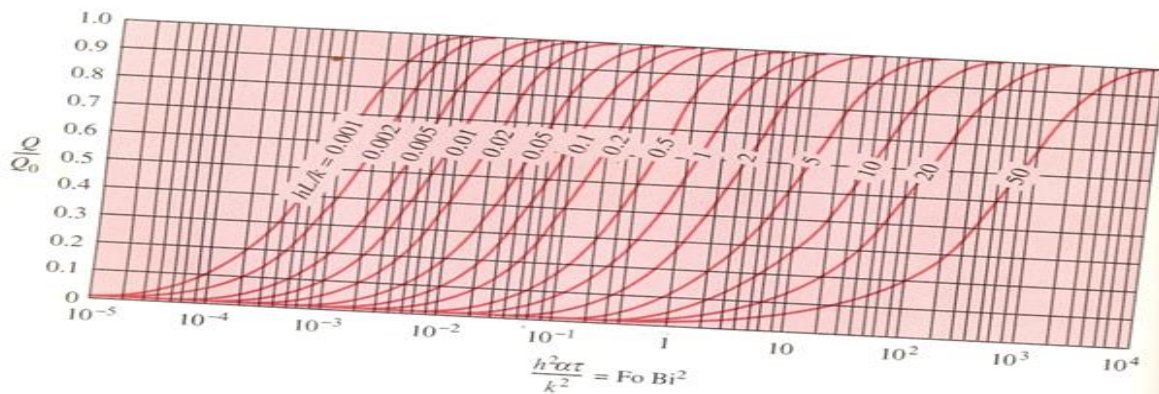
Acknowledgements:

First and foremost I want to thank my God, who gives me full health to done my work. I would like also to convey my deepest gratitude to my colleagues Dr Habtamu Itefa. It has been an honor to be his colleagues. I appreciate all his contributions of time, challenging ideas, critical remarks, constant guidance and support to make my article of journal and having experience productive and stimulating. My special thanks to you for providing me all the theoretical insight for many problems and especially for allowing me knocking your door without appointments.

Appendices:



Midplane temperature for an infinite plate of thickness 2L: (a) full scale.



Dimensionless heat loss Q/Q_0 of an infinite plane of thickness $2L$ with time, from Ref. 6.

References:

1. Tuomas laine, reijo hänninen and antti karola ,Benefits of BIM in the thermal performance management, Olof Granlund Oy, Malminkaari 21, 00700 Helsinki, Finland
2. Jaan Kiusalaas ,Numerical methods in engineering with python from page (190-333)
3. William F. Ames. *Numerical Methods for Partial Differential Equations*.Academic Press, Inc., Boston, third edition, 1992.
4. Hui, S. C. M., 2000. Building energy efficiency standards in Hong Kong and mainland China, In *Proc. of the 2000 ACEEE Summer Study on Energy Efficiency in Buildings*, 20-25 August 2000, Pacific Grove, California
5. M. Hazami*, S. Kooli, M. Lazâari, A. Farhatı and A. Belghith2, Energy and exergy efficiency of a daily heat storage unit for buildings heating, *Revue des Energies Renouvelables Vol. 12 N°2 (2009) 185 – 200*
6. Avadaiaappa P. Pasupathy1 and Ramalingom Velraj,Mathematical Modeling and Experimental Study on Building Ceiling Using Phase Change Material for Energy Conservation,*The 2nd Joint International Conference on "Sustainable Energy and Environment (SEE 2006)"21-23 November 2006, Bangkok, Thailand*
7. Piia Lamberg a, Reijo Lehtiniemi b, Anna-Maria Henell b, "Numerical and experimental investigation of melting and freezing processes in phase change material storage", *International Journal of Thermal Sciences* 43 (2004) 277-287).
8. Won Young Yang,Wenwu Cao,Tae-Sang Chung,John Morris,"APPLIED numericalmethods using matlab", a john wiley & sons, inc., publication
9. National Meteorological Agency Agrometeorological Bulletin ,"Seasonal Agrometeorological Bulletin", "Bega 2010/2011 Volume 21 No. 3"
10. Atul Sharma a,* , V.V. Tyagi b, C.R. Chen a, D. Buddhi b,Review on thermal energy storage with phase change materials and applications, *Renewable and Sustainable Energy Reviews* 13 (2009) 318-345